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to its progress. It is the nature of a textbook to reflect the integrated ideas of many individuals over more than century, so only a few of the many contributors are recognized by citation. Even so, a wealth of additional material is available to the reader, and that material provides a much more complete picture. We have included a few citations in the text on particular points and at the end of each chapter as additional material, so that the student has a entryway to the extensive library of published work that now is available.

The revisions also include many corrections and focused responses to suggestions received from colleagues, readers elsewhere, and especially from our students. We hope they will find the revisions to their liking. For the future we continue to invite comments and criticisms from students and faculty colleagues.

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