



Introduction: why and how to use this book

The study of biology in the nineteenth century was largely concerned with definition, description, and classification: collecting specimens, naming them, and arranging them in display cabinets, or drawing detailed anatomical diagrams and labelling them. Detailed description is still an important aspect of some fields of modern biosciences – cell biology, for example – but biology in the twenty-first century is all about **measuring and modelling change and development**, both of individual organisms and of populations. Because of this, mathematical and statistical techniques are now essential tools for the bioscientist.

There is a field of mathematics called the **calculus**, which is used to analyse rates of change. The second part of this book explains the calculus and shows how it is used in the biosciences. The first part of the book establishes the mathematical foundations needed for the calculus (what is referred to in the US as pre-calculus), though these foundations are also important for scientists in their own right. The principal foundation is the concept of a **function**; functions can be used to model the way a physical quantity changes depending on factors such as temperature and time (for example, the change in blood sugar levels as time passes after a meal).

We will use three modelling case studies, with instalments appearing in boxes throughout the book:

- Case Study A: Models of population growth
- Case Study B: Models of cancer
- Case Study C: Predator–prey relationships

These case studies will be used to illustrate the mathematical techniques of each chapter. They all involve real, practical biology, and though Case Study B initially looks more complex and difficult than Case Study A, and Case Study C is easier, they will all be using essentially the same mathematical concepts.

But this book is more than a summary of the mathematics taught at school or college. It is intended for students of the biosciences at both undergraduate and postgraduate degree level, and I will be presenting the mathematics from a university-level viewpoint also. My aim is not just to give you the rules and techniques you'll need to use, for you simply to memorize parrot-fashion, but to explain *why* they work, and how everything fits together consistently and elegantly. In this way, maths should not seem like a bewildering array of arbitrary rules. A wider aim is to convince you that mathematics can be fun, exciting, and beautiful, and is not just the domain of geeks and eggheads. I'm not referring here to mathematical puzzles and diversions, but the maths that can provide insight into the miracles and mysteries of

Nature. To this end, the final chapter of this book introduces concepts of dynamical systems and chaos theory, which is a field of much current research at the interface of mathematics, computing, science, and engineering.

The scope of this book covers 'only' mathematics, and not probability or statistics. There is one exception: we will give the mathematical derivation of the best-fit straight line through a set of data-points (Section 8.7), as this is a natural extension to finding the equation of a straight line through two points, and illustrates some important mathematical techniques. For a primer on the use of statistics in the biosciences, see the OUP companion volume *Biomeasurement: A Student's Guide to Biological Statistics* (Hawkins, 2005).

There are plenty of biological examples in the book, intended to be accessible to students of all fields in the life sciences. But I should emphasize that this is a mathematics textbook, not a bioscience book. Its structure is intended to build up all the mathematics systematically. Mathematics is a very linear subject: you need to know the rules of arithmetic in order to learn to use algebra; you can't handle functions without algebra; you need to know about functions in order to apply the ideas of calculus; and you can then use calculus to solve differential equations. By contrast, in biology you can study animal behaviour without knowing about the structure and function of the animal's internal organs, or the biology of the cells that make up those organs.

We will start with the basics of addition and subtraction, and present a self-contained body of mathematics which will lead to the solution of systems of differential equations (as used in epidemiology and predator-prey models). Important concepts, such as the roots of a function, may be defined more than once in different chapters, to cater for students who skip sections they believe they are familiar with. I would encourage you to at least skim through such sections, especially Chapter 1, where you will see the rules of arithmetic and algebra, which were shown to you piecemeal over a decade of schooling, used to construct what I would claim is humankind's greatest achievement – the Real Number system. Each chapter is intended to present something new and useful, whether you stopped studying maths at GCSE or went on to Maths or even Further Maths at A-Level.

As well as the algebra you will recall from school, the emphasis in this book (and particularly in the Extension sections) is on **computation**: techniques for finding numerical solutions to problems. As the quotation from *The Hitchhiker's Guide to the Galaxy* (Adams 2009) illustrates, even the most profound questions about the natural world can have a numerical solution! We'll see how numerical algorithms (sets of instructions, like computer programs) can be used to solve complex practical problems through a series of simple calculations. These numerical techniques are covered mainly in the **Extension sections** at the end of most chapters. These are not 'extensions' in the sense of being optional extras which only the mathematically gifted student would understand. On the contrary, I have found that even bioscience students with a positive aversion to maths will appreciate and enjoy applying these techniques, armed with a calculator and Excel. They are also techniques of increasing practical importance; in fact, reviewers of the book drafts have said that they are particularly important for today's bioscientists.

I won't be teaching you computer programming in this book. To perform calculations I'll assume that you have an electronic calculator, and you'll see that even a simple calculator can do more than you might expect. The other computing tool we'll use is **Microsoft Excel**.²

² Excel is a trademark of the Microsoft group of companies. *Core Maths for the Biosciences* is an independent publication and is not affiliated with, nor has it been authorized, sponsored, or otherwise approved by Microsoft Corporation.

MS Excel (which we'll refer to henceforth simply as Excel) is a spreadsheet package. A spreadsheet is a grid of cells; each cell can be used either to store text or a number, or it can be programmed to make calculations using data from other cells. Other spreadsheet programs are available, which have very similar features, but Excel is by far the most widely used and best-known program (being part of the Microsoft Office suite). You have probably already learned how to use Excel at school or college. I'll introduce the features of Excel which we'll need as we go along, but it would be good if you have a basic prior knowledge. As well as showing you how to construct your own spreadsheets, I'll give you ready-made spreadsheets as 'numerical laboratories' to allow you to experiment with the mathematical concepts. (See the **Technical Notes** section below.)

Excel can be used to plot graphs from data, but for sketching the graph of a function we'll mainly use a Freeware package called **FNGraph**. I am very grateful to Dr Alexander Minza for permission to use the package, by including screenshots as diagrams in the book, and providing FNGraph files on the website. There is an introduction to FNGraph in Chapter 6.

The Online Resource Centre that accompanies this book (<http://www.oxfordtextbooks.co.uk/orc/reed/>) has downloads of all the Excel and FNGraph files, as well as the FNGraph program itself; see the **Technical Notes** section which follows. You'll have to obtain your own copy of Excel, I'm afraid. The Online Resource Centre also has solutions to problems and web pages with supplementary content (including external links). To get the most from this book, you should work with a calculator and computer, and open the Excel and FNGraph files when they are referred to in the text.

Mathematics is a way of thinking, and this can be practised in many different contexts. So the Problems include applications of maths in other areas, which I hope you'll find interesting. If you can work through all of the problems, I hope that you'll not only be able to apply maths in your degree course, but you'll be thinking like a mathematician. On that scary note, I'll leave you to get started with Chapter 1 ...

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I would welcome any comments about the book, to help improve future editions, and would especially appreciate reports of any misprints or errors. Send your comments via <http://www.oup.com/uk/orc/feedback/> or follow the 'Send us your feedback' link on the left of the main Online Resource Centre page.