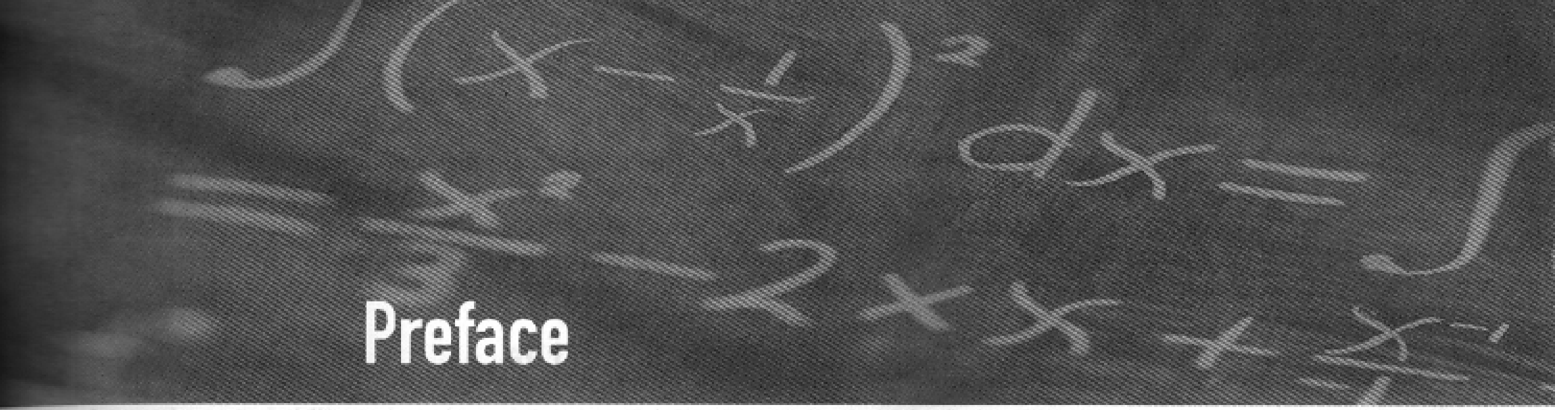

Preface

Calculus for the Life Sciences: Modelling the Dynamics of Life is truly a *calculus* book for *life science* students.

This book covers limits, continuity, derivatives, integrals, and differential equations, which are standard topics in introductory calculus courses in universities. All concepts, definitions, and theorems are explained in detail and illustrated in a number of fully solved examples. A variety of approaches—algebraic, geometric, numerical, and verbal—facilitate the understanding of the material and will be of great help to any students who try to read the book on their own.

As new mathematics concepts and ideas are introduced, applications illustrating their use are presented. Questions arising from life sciences situations are employed to motivate the construction of several mathematical objects (such as derivatives and integrals). A narrative introduces the context of an application and shows its relevance and importance in life sciences.

The major purpose of this book is to build *quantitative skills* and to introduce its readers to the *insights* that mathematics can provide into all branches of life sciences.

Although the importance of quantitative skills in the life sciences is a widely accepted fact, realities—in many cases—conceal their vital role. In part, this is because mathematics tends to be hidden, working in the background: once something has been figured out and becomes a standard, the mathematics that was used is no longer needed, as practitioners use software, diagrams, and charts instead. For instance, doctors and nurses do not look at half-life information to calculate a dosage every time they need to administer a medication. An ultrasound technician can "calculate" the volume of blood in a patient's heart chamber by selecting a few items from the pull-down menus in a computer program, completely unaware of the fairly sophisticated mathematics (calculation of volume by approximate integration) used to design the software.

As for the insights, a simple mathematical model explains why the heart of a mouse beats about 15 times as fast as the heart of an elephant. Knowing mathematical formulas that describe human growth, we can understand why body mass index does not give reliable estimates of body fat in children and tall people.

Furthermore, mathematics has the power to *reveal otherwise invisible worlds* in the vast universe of numerical data that has been gathered about almost every single phenomenon related to life. Joel E. Cohen writes:[1]

> For example, computed tomography can reveal a cross-section of a human head from the density of X-ray beams without ever opening the head, by using the Radon transform to infer the densities of materials at each location within the head. Charles Darwin was right when he wrote that people with an understanding "of the great leading principles of mathematics . . . seem to have an extra sense." Today's biologists increasingly recognize that appropriate mathematics can help interpret any kind of data.

Although a great deal of biology can be done without mathematics, the powerful new technologies that are transforming fields of biology—from genetics to physiology

[1] J. E. Cohen. Mathematics is biology's next microscope, only better; biology is mathematics' next physics, only better. PLoS Biol 2(12): e439, 2004. Published online December 14, 2004. doi: 10.1371/journal.pbio.0020439. PMCID: PMC535574.

to ecology—are increasingly quantitative, as are many questions at the frontiers of knowledge. Along with genetics, mathematics is one of two unifying factors in the life sciences. And as biology becomes more important in society, mathematical literacy becomes as vital for an informed citizen as it is for a researcher.

Modelling and the Dynamics of Life

The goal of this book is to teach the mathematical ideas that will help us understand various phenomena in life sciences. These are the same ideas that researchers use in their work, as well as in collaborations with colleagues engaged in more empirical activities. They are not specific techniques, such as differentiation or integration by parts, but rather they revolve around building *mathematical models*.

A mathematical model is that crucial link between a life sciences phenomenon and its description in terms of mathematical objects. We will gain the skills needed to construct a model, make sure it works, and understand what it implies—we will learn how to *translate* appropriate aspects of a life sciences problem and its assumptions into formulas, equations, and diagrams; how to *solve* the equations involved; and how to *interpret* the results in terms of the original problem. For instance:

- We build several models in an attempt to predict what the population of Canada will be in the near (and not so near) future. In each case, we critically examine the assumptions that we made, as well as comment on the results of the model.
- Regular clinical breast examinations are key to early detection of breast cancer. Do these exams suffice, or should one also consider mammography? By using an exponential model to understand how cancer cells grow, we conclude that in some cases mammography gives a significant lead time over clinical examination in early detection of cancer.
- To study the interaction between populations, we take a detailed look at the predator-prey model, which could be used, for instance, to describe the interaction between foxes and rabbits in an ecosystem. A model of selection will help us understand the behaviour of two different bacterial cultures sharing the same space and resources.
- We investigate limited population growth models, such as the logistic model and the Allee Effect.
- Using discrete-time dynamical systems, we build our understanding of aspects of consumption and absorption of drugs such as alcohol and caffeine.

Content

Mathematics helps us unify biology by identifying principles that underlie a remarkable diversity of biological processes. This book follows three themes throughout: growth, diffusion, and selection. Each theme is studied with two kinds of models: *discrete-time dynamical systems* and continuous processes that use *differential equations*. These themes provide a logical context to teach the standard calculus material.

First, we outline details of a mathematical model and review basic functional building blocks (such as graphs and transformations of graphs, composition of functions, and inverse functions). In the Introduction and Chapter 1 we review facts about elementary functions (algebraic and transcendental) and, at the same time, discuss numerous models based on these functions. For instance, using power functions we study heartbeat frequency in mammals and derive an important relationship between the size of an animal and the size of its body cover. One of the models for the population of Canada involves exponential functions. Another application involves cell growth.

Next, in Chapter 2, we introduce discrete-time dynamical systems and their applications. Throughout the book, as we learn more math, we come back to these systems and enrich our understanding of their basic features (such as stability). Three sections in this chapter are fully devoted to studying applications: gas exchange in lungs, the dynamics of the consumption and absorption of drugs, and a nonlinear model of selection.

Chapter 3 contains a discussion of major ideas in calculus: limits, continuity, and derivatives. In Chapters 4 and 5 we learn how to calculate derivatives and how to use them in both mathematics and applications. The book emphasizes *reasoning about functions*: it includes sections devoted to applications of such general theorems as the Intermediate Value Theorem and the Mean Value Theorem, sketching graphs using derivatives and the method of leading behaviour, and approximating functions using Taylor polynomials. These skills are then applied to reasoning about the stability of dynamical systems and about various aspects of solutions of differential equations.

In Chapter 6 we discuss antiderivatives, and construct definite integrals, investigate their meaning, and articulate their connection in the statement of the Fundamental Theorem of Calculus. We learn basic integration methods and study applications, including area and average value. As an illustration of the concept of a Riemann sum we attempt to obtain a good estimate of the surface area of Lake Ontario.

In Chapter 7, the last chapter, we study differential equations, with special emphasis on autonomous differential equations and their applications (logistic growth, law of cooling, diffusion across a membrane, continuous model of selection). Although we do explain how to solve (some) differential equations algebraically, we focus on qualitative analysis (stability of equilibria, graphical display). The chapter ends with the most challenging material in the book: analysis of systems of differential equations representing the predator-prey model and the dynamics of competition. Again, we focus on qualitative features such as equilibria, nullclines, and direction fields.

Presentation

Easy-to-read and easy-to-follow narratives, carefully drawn pictures and diagrams, clear explanations, large numbers of fully solved examples, and broad-spectrum applications make the material suitable for a variety of audiences with a wide range of interests and backgrounds.

In creating this book, we were guided by several important principles:

- Convey excitement about the material.
- Convey the relevance and importance of mathematics and its applications.
- Use a variety of approaches—algebraic, numerical, geometric, and verbal.
- Motivate the introduction of new mathematical objects, concepts, and algorithms.
- Review background material so that it does not become an obstacle in explaining advanced material.
- Provide more examples and illustrations for more difficult material.
- Revisit important concepts as often as possible, in a variety of contexts.
- Revisit applications and enrich models as new mathematics ideas are introduced.

Conceptual Understanding and Mathematical Thinking

This text provides the reader with an opportunity to build a clear understanding of a relatively small number of ideas and concepts from the calculus of functions of one variable. It offers exhaustive discussions and carefully crafted examples; clear and crisp statements of theorems and definitions; and—acknowledging that many of us are visual learners—numerous illustrations, graphs, and diagrams. Important concepts are revisited as often as possible, placed in a variety of contexts, and applied in solving life sciences problems.

Development of Mathematical (Quantitative) Skills

It is impossible to fully master almost any topic in mathematics without adequate skills in symbolic (algebraic) manipulation. This book contains a large number of fully solved examples designed to illustrate formulas, algebraic methods, and algorithms, and to provide an ideal opportunity for students to improve on their routine in the technical intricacies of calculations.

Topics that some students may find challenging (such as Riemann sums and integration methods, working with Taylor polynomials, and graphing using leading behaviour) are accompanied by a large number of solved examples. Use of technology (graphing calculator or mathematical software such as Maple) is strongly encouraged, since it might provide insights that will enhance understanding of the material.

In-Depth Explorations of Particular Models

This book includes several extended applications. Early on, the power of cobwebbing is put to work to study the phenomenon of the interaction between two types of bacteria (mutant and wild; Section 2.5). The applications of maximization to a food intake problem and to optimizing the work of the heart are presented in Section 5.1. The techniques of optimization in interaction with dynamics are employed to study optimal breathing patterns in Section 5.7. Phase-plane methods are used in Chapter 7 to analyze two-dimensional systems representing interactions between two species. Some of these models can be assigned as individual or group study projects.

End-of-Section and End-of-Chapter Problems, Computer Exercises, and Projects

In addition to routine skills problems, each section includes a wide variety of modelling problems to emphasize consistently the importance of *interpretation.* As well, we will find more challenging mathematics questions here, including requests to construct proofs that are omitted from the text. Each chapter includes supplementary problems that introduce a variety of new applications and can be used for review or practice sessions.

The book contains over 50 exercises designed to be explored on a graphing calculator (preferably programmable) or on a computer using software such as Maple. These exercises emphasize *visualization, experimentation,* and *simulation*—all of which are important aspects of conducting research in life sciences, as well as many other areas.

Each chapter includes projects suitable for individual or group exploration. Examples include the following:

- modelling the balance between selection and mutation (Chapter 2)
- studying periodic hematopoiesis using a discrete-time dynamical system (Chapter 3)
- experimenting with different numerical schemes for solving differential equations (Chapter 6)
- carefully studying models of adaptation by cells (Chapter 7)

Teaching

Although this is a book for *life sciences*, it is a *calculus textbook* as well. It can be used to teach different courses, such as

- a one-semester calculus course of a more theoretical nature (i.e., focusing on mathematical ideas, theory, and algorithms—all of which are covered in this book, including proofs—and perhaps occasionally covering an application)
- a one-semester, first-year calculus for life sciences course (with the possible omission of certain material, such as two-dimensional dynamics in Chapter 7 and full-section-length applications)
- a second-year modelling course for students who have taken traditional calculus (emphasis on discrete-time dynamical systems in Chapter 2, applications of derivatives in Chapter 5, differential equations in Chapter 7, and full-section-length applications)

Great care has been taken to make the level of exposition—both mathematical and applications—adequate for first-year students. A course instructor can assign, with confidence, parts of the material as homework or as optional reading.

No matter which course is taught from this text, the benefits are obvious. *The modelling approach is naturally a problem-solving approach.* Students will not remember every technique they have learned. This book emphasizes understanding what a model *is* and recognizing what a model *says.* To be able to recognize a differential equation, interpret the terms, and use the solution is far more important than knowing how to find a solution algebraically. These reasoning skills, in addition to familiarity with the models in general, are what will stay with the motivated student and what will matter most in the end.

Instructor Resources

Instructor's Resource CD
ISBN 0176441967
The *Instructor's Resource CD for Calculus for the Life Sciences*, First Canadian Edition, holds all instructor resources in one location, so there is no hassle with multiple media files or disks. The CD contains

- a NETA ExamView® computerized test bank and NETA Test Bank in rich-text format (see below for a description of NETA)
- the full Instructor's Solutions Manual, with worked-out solutions for each exercise in the text, prepared by Miroslav Lovrić, author of the Canadian editon of the texbook.

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NELSON EDUCATION TEACHING ADVANTAGE

Introducing the Nelson Education Teaching Advantage (NETA)
In most college and university courses, a large percentage of student assessment is based on multiple choice testing. But many instructors use multiple choice reluctantly, believing that it is a methodology best used for testing what a student remembers rather than what she or he has learned. Furthermore, the quality of publisher-supplied test banks can vary. Nelson Education Ltd. believes that a good-quality multiple choice test bank can test not just what students remember, but higher-level thinking skills as well.

Recognizing the importance of multiple choice testing in today's classroom, Nelson has created the Nelson Education Teaching Advantage program to ensure the high quality of our test banks.

NETA was created in partnership with David DiBattista, a 3M National Teaching Fellow and professor of psychology at Brock University. NETA ensures that test bank authors have had training in two areas: developing clear multiple choice test questions while avoiding common errors in construction, and creating multiple choice test questions that "get beyond remembering" to assess higher-level thinking.

The outcome of NETA development is that as you select multiple choice questions from your Nelson test bank for inclusion in tests, you can easily identify whether items are memory-based or require your students to engage in higher-level thinking. By making your selections appropriately, you can construct tests that contain the proportion of recall and higher-level questions that reflects your personal instructional goals. All NETA test banks are accompanied by David DiBattista's guide for instructors, "Multiple Choice Tests: Getting Beyond Remembering." This guide has been designed to assist you in using Nelson test banks to achieve your desired outcomes in the classroom.

The NETA test materials developed for *Calculus for the Life Sciences* are presented in two formats:

Test Bank: This thoroughly revised Test Bank was created by Alan Ableson of Queen's University. Every question and answer was read and reviewed for accuracy by multiple sources. A Canadian proofreader checked all questions, including the new or revised questions created for the First Canadian Edition. Each chapter of the test bank is organized by type of question and includes text page references. Files are provided in rich text format for easy editing and printing with all common word-processing formats.

Computerized Test Bank: All Test Bank questions are included in the ExamView computerized version. The easy-to-use software is compatible with Microsoft Windows and Mac. Create tests by selecting questions from the question bank, modifying these questions as desired, and adding new questions you write yourself. You can administer quizzes online and export tests to WebCT, Blackboard, and other formats.

Student Resources

Enhanced WebAssign®

Enhanced WebAssign is a powerful instructional tool that delivers automatic grading of solutions for math courses, and reinforces student learning through practice and instant feedback. This proven and reliable homework system allows instructors to assign, collect, grade, and record homework assignments via the Web. Contact your Nelson sales representative for more information.

Student Solutions Manual

ISBN 0176441786

The *Student Solutions Manual* contains detailed solutions to all odd-numbered end-of-section exercises and end-of-chapter supplemental problems. It was prepared by Miroslav Lovrić, McMaster University, and technically checked by Caroline Purdy, University of New Brunswick.

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Miroslav Lovrić
Hamilton, 2011