Preface

Biological invasion is one of the most challenging and important issues in contemporary ecology. Patterns of species spread, rates of spread, the impact of various biological and environmental factors and other related problems have been under intensive study for a few decades. New effective tools and approaches have been developed, important work has been done and considerable progress has been made towards better understanding of this phenomenon.

Although a lot of results regarding biological invasion were obtained through field studies and analysis of field observations, recent advances could hardly be possible without extensive use of mathematics, in particular, mathematical modeling. The reason for this has its roots in the very nature of the problem. A regular study based on manipulated field experiments is very difficult due to the virtual impossibility of reproducing the environmental and initial conditions. Laboratory experiments are often not effective due to inconsistence of spatial scales. In these situations, mathematical modeling takes, to some extent, the role that is normally played by experimental study in other natural sciences.

It should also be mentioned that the issue of biological invasion has been an inspiration for a few generations of mathematicians. Starting from classical works by Fisher (1937) and Kolmogorov et al. (1937), this subject has been fascinating ever since and eventually became one of the cornerstones for the contemporary nonlinear science.

However, the whole understanding of the meaning and implication of mathematics in scientific studies has been changing recently. Due to tremendous progress in computational technologies in the last two decades, many theoreticians working in various fields tended to consider computer experiment as their main research tool. The whole concept of mathematical modeling was reduced to numerical simulations and the role of rigorous mathematics has often been underestimated. It must be noted that, although computer experiment is a useful and powerful approach, it has a few serious drawbacks and its actual capability is not as almighty as it is sometimes viewed. One of the drawbacks is the impact of numerical/approximation error which is usually very difficult to estimate and which, in applied numerical simulations, is practically never addressed so that its actual impact on simulation results in most cases remains obscure. Meanwhile, there are examples when the approximation error changes results essentially, especially when the problem under study is nonlinear. Another drawback is that, since even a very detailed simulation study uses a finite number of parameter values, computer experiment alone is not capable of providing full and complete information about the solution dependence on the parameters. The information obtained numerically is more meager the more parameters the problem depends on.

These difficulties never arise when the problem can be solved analytically. It should be mentioned that, when a nonlinear problem is described by partial differential equations (PDEs), the exact solution is usually a special solution, i.e., the solution possessing certain symmetry and/or obtained for special initial and boundary conditions. (Probably the most well-known example of a special solution with an immediate application to biological invasion is a traveling population front.) Remarkably, however, the meaning of such solutions is not exhausted by a few special situations and appears much more general due to initial conditions convergence which is often considered as a manifestation of a universal principle of scaling.

Besides providing an immediate and complete description of system dynamics for a relevant class of initial conditions (apart from an early stage of dynamics when the initial details can be significant), exact solutions can also infer information about a larger family of related problems via application of the comparison principle. Another useful application of exact solutions which should not be forgotten is that they serve as a convenient tool for testing complicated numerical algorithms and codes used in more specialized studies.

In this book, our main attention is given to the models based on nonlinear PDEs, especially PDEs of diffusion-reaction type. One of the reasons why the implication of exactly solvable nonlinear models in ecology has been underestimated is the widely spread opinion that such models are exceptionally rare. This is not true: indeed, they are relatively rare but not at all exceptional. Along with several ad hoc methods, there are some regular mathematical approaches that can be applied to obtain exact solutions of nonlinear PDEs. One of the goals of this book is to provide a unified description of these methods in order to bring them into use by a wider community of theoreticians working on species dispersal and biological invasion. Apart from the methods and relevant examples, we also give a review of exactly solvable models (now scattered over periodic literature) that can be useful for studying biological invasion and species spread.

Although this book is mostly concerned with mathematical aspects of invasive species spread, we never forget that there is nature standing behind. Based on exactly solvable models, we make a new insight into a few issues of significant current interest such as the impact of the Allee effect, the impact of predation, the interplay between different modes of species dispersal, etc. Thus, we expect that our book will be interesting and useful not only to applied mathematicians but also to those biologists who are not scared by differential equations. In order to make it intelligible to researchers from different fields as well as to postgraduate students, we provide as many calculation details as possible. Also, an Appendix is added giving a brief review of some background mathematics. The chapters that contain a somewhat more complicated mathematics and can be skipped for the first reading are marked with asterisk.

The book is organized as follows. In Chapter 1, we begin with providing arguments to show why exactly solvable models are important. In particular, we give a few examples when a straightforward numerical study of relevant ecological problems can be misleading. We then give a brief review of basic facts from population dynamics that should be taken into account when formulating mathematical models of biological invasion. In Chapter 2, we give an overview of several modeling approaches that have been developed and successfully applied over the last two decades. We briefly discuss the biological background behind different approaches in order to better understand their applicability and try to reveal mathematical relation between different models. In Chapter 3, we describe a few methods that are used to construct exact solutions of nonlinear diffusion-reaction equations and consider a few instructive examples. In Chapters 4 to 6, we consider several more specific ecological problems described by exactly solvable models. In Chapter 7, we show how the predictions of exactly solvable models can sometimes be extended beyond their presumptions and what alternative analytical approaches are possible in non-integrable cases. In Chapter 8, we demonstrate the usefulness of exactly solvable models by means of applying them to a few particular cases of biological invasions. Finally, the last chapter gives some basic facts about the mathematical tools used throughout the book.

This preface would not be complete if we did not mention those who have greatly helped us in our work. We think that this is a wonderful opportunity to express our sincere gratitude to the people who were our teachers at different stages of our life and whose influence on our research cannot be overestimated. We are especially thankful to Grigorii Barenblatt, Simon Levin, Nanako Shigesada and Wally Wu who gave us timeless ideas about the true standards of scientific study. We are very thankful to our colleagues and friends who throughout all the years have collaborated with us, including Horst Malchow, Rod Blackshaw, Ezio Venturino, Sasha Medvinsky, Zhen-Shan Lin, Andrew Morozov, Frank Hilker and Igor Nazarov. Many thanks are also due to Britta Daudert for her careful reading of the complete draft. We are very grateful for the valuable support and patience of our families; they understood and shared our joy in writing this book. Last but not least, we are thankful to Sunil Nair, Publisher of Mathematics and Statistics, CRC Press/Chapman & Hall, for inviting us to write this book and for his patience and encouragement.

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