

Preface

The theory of one-parameter semigroups of linear operators on Banach spaces started in the first half of this century, acquired its core in 1948 with the Hille–Yosida generation theorem, and attained its first apex with the 1957 edition of *Semigroups and Functional Analysis* by E. Hille and R.S. Phillips. In the 1970s and 80s, thanks to the efforts of many different schools, the theory reached a certain state of perfection, which is well represented in the monographs by E.B. Davies [Dav80], J.A. Goldstein [Gol85], A. Pazy [Paz83], and others.

Today, the situation is characterized by manifold applications of this theory not only to the traditional areas such as partial differential equations or stochastic processes. Semigroups have become important tools for integro-differential equations and functional differential equations, in quantum mechanics or in infinite-dimensional control theory. Semigroup methods are also applied with great success to concrete equations arising, e.g., in population dynamics or transport theory. It is quite natural, however, that semigroup theory is in competition with alternative approaches in all of these fields, and that as a whole, the relevant functional-analytic toolbox now presents a highly diversified picture.

At this point we decided to write a new book, reflecting this situation but based on our personal mathematical taste. Thus, it is a book on semigroups or, more precisely, on one-parameter semigroups of bounded linear operators. In our view, this reflects the basic philosophy, first and strongly emphasized by A. Hadamard (see p. 152), that an autonomous deterministic system is described by a one-parameter semigroup of transformations.

Among the many continuity properties of these semigroups that were

already studied by E. Hille and R.S. Phillips in [HP57], we deliberately concentrate on strong continuity and show that this is the key to a deep and beautiful theory. Referring to many concrete equations, one might object that semigroups, and especially strongly continuous semigroups, are of limited value, and that other concepts such as integrated semigroups, regularized semigroups, cosine families, or resolvent families are needed. While we do not question the good reasons leading to these concepts, we take a very resolute stand in this book insofar as we put strongly continuous semigroups of bounded linear operators into the undisputed center of our attention. Around this concept we develop techniques that allow us to obtain

- a semigroup on an appropriate Banach space even if at first glance the semigroup property does not hold, and
- strong continuity in an appropriate topology where originally only weaker regularity properties are at hand.

In Chapter VI we then show how these constructions allow the treatment of many different evolution equations that initially do not have the form of a homogeneous abstract Cauchy problem and/or are not “well-posed” in a strict sense.

Structure of the Book

This is not a research monograph but an introduction to the theory of semigroups. After developing the fundamental results of this theory we emphasize spectral theory, qualitative properties, and the broad range of applications. Moreover, our book is written in the spirit of functional analysis. This means that we prefer abstract constructions and general arguments in order to underline basic principles and to minimize computations. Some of the required tools from functional analysis, operator theory, and vector-valued integration are collected in the appendices.

In Chapter I, we intentionally take a slow start and lead the reader from the finite-dimensional and uniformly continuous case through multiplication and translation semigroups to the notion of a strongly continuous semigroup.

To these semigroups we associate a generator in Chapter II and characterize these generators in the Hille–Yosida generation theorem and its variants. Semigroups having stronger regularity properties such as analyticity, eventual norm continuity, or compactness are then characterized, whenever possible, in a similar way. A special feature of our approach is the use of a rich scale of interpolation and extrapolation spaces associated to a strongly continuous semigroup. A comprehensive treatment of these “Sobolev towers” is presented by Simon Brendle in Section II.5.

In Chapter III we start with the classical Bounded Perturbation Theorem III.1.3, but then present a new simultaneous treatment of unbounded Desch–Schappacher and Miyadera–Voigt perturbations in Section III.3. In the remaining Sections III.4 and 5 it was our goal to discuss a broad range

of applications of the Trotter–Kato Approximation Theorem III.4.8.

Spectral theory is the core of our approach, and in Chapter IV we discuss in great detail under what conditions the so-called spectral mapping theorem is valid. A first payoff is the complete description of the structure of periodic groups in Theorem IV.2.27.

On the basis of this spectral theory we then discuss in Chapter V qualitative properties of the semigroup such as stability, hyperbolicity, and mean ergodicity. Inspired by the classical Liapunov stability theorem we try to describe these properties by the spectrum of the generator. It is rewarding to see how a combination of spectral theory with geometric properties of the underlying Banach space can help to overcome many of the typical difficulties encountered in the infinite-dimensional situation.

Only at the end of Chapter II do differential equations and initial value problems appear explicitly in our text. This does not mean that we neglect this aspect. On the contrary, the many applications of semigroup theory to all kinds of evolution equations elaborated in Chapter VI are the ultimate goal of our efforts. However, we postpone this discussion until a powerful and systematic theory is at hand.

In the final chapter, Chapter VII, Tanja Hahn and Carla Perazzoli try to embed today's theory into a historical perspective in order to give the reader a feeling for the roots and the *raison d'être* of semigroup theory.

Furthermore, we add to our exposition of the mathematical theory an epilogue by Gregor Nickel, in which he discusses the philosophical question concerning the relationship between semigroups and evolution equations and the philosophical concept of "determinism." This is certainly a matter worth considering, but regrettably not much discussed in the mathematical community. For this reason, we encourage the reader to grapple and come to terms with this genuine philosophical question. It is enlightening to see how such questions were formulated and resolved in different epochs of the history of thought. Perhaps a deeper understanding will emerge of how one's own contemporary mathematical concepts and theories are woven into the broad tapestry of metaphysics.

Guide to the Reader

The text is not meant to be read in a linear manner. Thus, the reader already familiar with, or not interested in, the finite-dimensional situation and the detailed discussion of examples may start immediately with Section I.5 and then proceed quickly to the Hille–Yosida Generation Theorems II.3.5 and II.3.8 via Section II.1. To indicate other shortcuts, several sections, subsections, and paragraphs are given in small print.

Such an individual reading style is particularly appropriate with regard to Chapter VI, since our applications of semigroup theory to the various evolution equations are more or less independent of each other. The reader should select a section according to his/her interest and then continue with the more specialized literature indicated in the notes. Or, he/she may

even start with a suitable section of Chapter VI and then follow the back references in the text in order to understand our arguments.

The exercises at the end of each section should lead to a better understanding of the theory. Occasionally, we state interesting recent results as an exercise marked by *.

The notes are intended to identify our sources, to integrate the text into a broader picture, and to suggest further reading. Inevitably, they also reflect our personal perspective, and we apologize for omissions and inaccuracies. Nevertheless, we hope that the interested reader will be put on the track to uncover additional information.

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