Preface

Since the first edition was published twenty years ago, a lot of theoretical results on geometric theory of membrane elasticity have been achieved. In particular, the shape equation and boundary conditions for open lipid membranes were obtained. The main changes in this second edition of the present book, we add a chapter (Chapter 4) to explain how to calculate variational problem on a surface with a free edge by using a new mathematical tool–moving frame method and exterior differential forms, and how to derive the shape equation and boundary conditions for open lipid membranes through this new method. In addition, we also include the recent concise work on chiral lipid membranes as a section in Chapter 5. In Chapter 6, we also mention some topics that we have not fully investigated but they are also important to geometric theory of membrane elasticity.

We owe our sincere gratitude to our colleagues such as Profs. Udo Seifert, Jemal Guven, Ivailo Mladenov, Qiang Du and so on since many supplemental results in this edition stem from their work. We also thank Pan Yang and Yang Wang for their patient help in typing this manuscript. Finally, Zhan-Chun Tu, Zhong-Can Ou-Yang and Ji-Xing Liu would like to dedicate this edition to Prof. Yu-Zhang Xie who passed away on May 29, 2011.

Preface for the First Edition

Liquid crystal state was discovered by F. Reinitzer in 1888. A review article written by G. Friedel in 1921 summarizes the general properties of liquid crystals known up to that time. In that article the correlation between liquid crystal and differential geometry was noticed for the first time. The focal conic texture of smectic liquid crystals was shown to have its geometrical origin from Dupin cyelides. However, for quite some time, geometrical studies on liquid crystals were in a stagnant state. Based on the structural similarity between smectic liquid crystal and fluid membrane, in 1973, W. Helfrich offered an elastic theory of membranes in analogy with the curvature elasticity theory of liquid crystals. Since then the study on the correlation between liquid crystals, biomembranes and differential geometry attracts great attention to both physicists and mathematicians. A review on the aspect of differential geometry in the study of biomembranes was given by mathematician J. C. C. Nitsch in 1993 [Q. Appl. Math. 51, 363 (1993)]. The present book is intended to serve as an introduction to those who are interested in this subject.

This book gives a comprehensive treatment on the conditions of mechanical equilibrium and the deformation of membranes as a surface problem in differential geometry. It is aimed at readers engaging in the field of investigation of the shape formation of membranes in liquid crystalline state with differential geometry. It does not offer a compiled survey of all the results obtained in this field. Since the pioneer publications by D. P. B. Canham and W. Helfrich on the geometrical form of membrane elasticity in the early 1970's, a lot of works on this cross disciplinary subject in physics, biology and mathematics have been published. The literatures concerning this field increase continuously. Many important results in which the shape problem of membrane vesicles being treated numerically have been published by authors such as W. Helfrich, H. Deuling, M. A. Peterson, S. Svetina, M. Wortis, R. Lipowski, U. Seifert, and others. However, under the personal interests of the present authors, the material chosen in this book is mainly limited in analytical results. They apologize to those authors whose beautiful results are not included here.

One of us (Z.-C. Ou-Yang) wishes to express his grateful thanks to Prof.

W. Helfrich for his kind guidance in leading him into this interesting field. We thank many colleagues for their collaborating research in this field. Special thanks are due to Profs. Peng Huan-wu, Hao Bai-Lin, Yu Lu and Su Zhao-Bin for their great support of our working in this cross-disciplinary field of theoretical physics and biology. Also we owe our sincere gratitude to many of our collaborators Profs. Zheng Wei-mou, H. Naito, S. Komura, only mentioned a few, with whom many results presented in this book were obtained. We thank Dr. Haijun Zhou for his patient help in typing this manuscript. We also thank World Scientific for waiting patiently for the long delayed submission of the final edition of the manuscript.

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December, 1998

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