

Preface

This book is intended to give an introduction to the theory of stochastic age-structured population dynamic system which has received strong attention in recent years because of its interesting structure and its usefulness in various applied fields.

The motivation for studying stochastic age-structured population dynamic system comes originally from biomathematics theory. Biomathematics, a newly emerged interdisciplinary subject, researches the quantitative feature and spatial pattern of life body and bio-system. Biomathematics also resolve problem of bioscience by theories and methods in mathematics. The 21st century is the century of bioscience, the century of information science and the century of microelectronics technology, which is also the century of all sciences gradual transform from quantitative research to qualitative research. With the features of bioscience and quantitative science, the basis of a great deal of information and computer technique as tool, the mathematical modeling of biological systems is evolving into an integral part of biological experiments.

The study of biomathematics started in early 20s, which was known as the golden age of theoretical ecologists solutions, so the discussion of the numerical method of stochastic population system to become the field of mathematical biological hot issue. The book mainly study the numerical method of the population with a variety of noise models. Including the author's research. The given numerical methods and the conclusions provide new ideas and theoretical basis for stochastic evolution-type partial differential equations numerical calculation, but also to the value of the stochastic population system provides a reliable method. Provide a strong basis to explore epidemics, ecological environment and the protection of the population. Since the dynamics method study of the life sciences was first proposed, and the logistic model, prey model and infectious diseases model was considered to be the most famous model. Subsequently, on the basis of the modeling, age structure, time-lag, migration, random interference of environment, intraspecific competition for resources, interspecific competition for resources have been considered. With the development of computer technology of the sixties and seventies years, the awareness of the seriousness of the ecological crisis to promote the further development of mathematical biology. To solve the five major worldwide problem: resources,

energy, environment, population and food are also related on Ecology. For example, to predict behavior of the system and make the best decisions of governance, also need the help of systems analysis and computer simulation. Thus, using computer simulation to study the complex ecosystem has become the main method of systems ecology research. The dynamic model of population and community has also been further developed, taking into account the detailed behavior of space consistency, stability, and vagueness, disagreement. Biomathematics' field of study is carried out on the human will face such as the lack of natural resources and energy, environmental pollution, overpopulation, disease and health is complex and difficult problem in the 21st century. Therefore, how to establish appropriate ecological model and numerical simulation are particularly important in the field of biomathematics.

In recent years, the study of taking into account the random disturbance on the population model system have been being attracted widely attention. In general, the majority of stochastic population system have no exact analytical solution. This book focuses on the latest research results of the numerical calculation of biomathematical research in stochastic population system, which is characterized by considering the effect of random noise to the the population system, and the numerical method and convergence is studied. To maintain systematic and easy to read, we first give the existence and uniqueness of the solution; followed by numerical calculation. Combined with their own research, by the main line to the value of single-species model, numerical methods of the various types of single-species model is clarified. Discuss the numerical calculation for stochastic population system with the Brownian motion, fractional Brownian motion, Markovian process and Poisson process.

The book is organized as follows. As an introduction, we present several theory of stochastic processes, Brownian motion, Itô's formula and basic inequalities in the first part. The second part have three chapters, which introduced populations system model systemically, study the existence and uniqueness of solutions of stochastic age dependent population. The third part of a total of seven chapters, focuses on the numerical calculation of stochastic age dependent population. According to the Euler method and the semi-implicit Euler method, stochastic age-dependent population was discussed, a sufficient condition for the convergence of the numerical approximation solution to the analytic solution and Exponential stability is given, and by the large number of numerical examples to verify the effectiveness of the algorithm, solving construct a stable algorithm for random population development system. There are three chapters of Part IV, mainly discussed the value of

stochastic delay neural networks.

We would like to thank all those who have helped us with their support and contributions in the preparation of this book. Special thanks are due to Professor Jin Zhen, Zhang Fengqin for their invaluable guidance and assistance. Special thanks are also due to our colleagues and friends in Ningxia University.

Zhang Qimin. would like to acknowledge partial support from the China National Science Foundation grant (Nos. 11061024, 11261043) and the Publishing Foundation Project of Ningxia University. Last but not least, we must express our special thanks to our family members for their love and endless encouragement and support.

1.2 Basic notations of probability theory	2
1.3 Stochastic processes	10
1.4 Brownian motions	12
1.5 Stochastic integrals	15
1.6 Itô's formula	31
1.7 Moment inequalities	40
1.8 Gronwall-type inequalities	45
Chapter 2 Existence, uniqueness and exponential stability for stochastic age-dependent population	45
2.1 Introduction	45
2.2 Assumptions and preliminaries	49
2.3 Existence and uniqueness of solutions	52
2.3.1 Uniqueness of solutions	52
2.3.2 Existence of strong solutions	53
2.4 Stability of strong solutions	59
Chapter 3 Existence and uniqueness for stochastic age-structured population system with diffusion	64
3.1 Introduction	64
3.2 Euler approximation and main result	65
3.3 Existence and uniqueness of solutions	68
3.3.1 Uniqueness of solutions	68
3.3.2 Existence of strong solutions	70
3.4 Numerical simulation example	75
Chapter 4 Existence and uniqueness for stochastic age-dependent population with fractional Brownian motion	79
4.1 Introduction	79
4.2 Preliminaries	81