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Appendix

A. General Results for the Laplacian Operator in Bounded Domains

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1.1 Continuous Growth Models

Single-species models are of relevance to laboratory studies in particular but, in the real world, can reflect a telescoping of effects which influence the population dynamics. Let $N(t)$ be the population of the species at time t , then the rate of change

$$\frac{dN}{dt} = \text{births} - \text{deaths} + \text{migration} \quad (1.1)$$

Kingland (1995) gives a fascinating historical and highly readable account of some of the major ideas introduced in the 20th century and of some of the scientists involved together with vignettes on how their own, often isolated, efforts affected the progress of the field.