

Contents

1	Introduction to Liquid Crystal Biomembranes	1
1.1	Liquid Crystals	1
1.1.1	Mysterious Matter	1
1.1.2	Orientational Order	3
1.1.3	Classification of Thermotropics	4
1.1.4	Classification of Lyotropics	7
1.2	Amphiphiles and Lyotropic Liquid Crystals	8
1.2.1	Amphiphile	8
1.2.2	Monolayer	9
1.2.3	Micelle	10
1.2.4	Phase Diagram	11
1.3	Phase Transitions in Biomembranes	12
1.3.1	Fluid Mosaic Model	12
1.3.2	Lipid Bilayer	13
1.3.3	Phase Transitions in Bilayer	14
1.3.4	Classification of Lipid Bilayer Phase	16
1.4	Simple Biochemistry of LC Biomembranes	18
1.4.1	Effect of Chain Length	18
1.4.2	Double Bond Effect	19
1.4.3	Effect of Ionic Condition	20
1.4.4	Cholesterol Effect	20
1.4.5	Complexity in Biomembrane	22
1.5	Artificial Bilayers and Vesicles	22
1.5.1	Lipids for Artificial Vesicles	23
1.5.2	Multilamellar Vesicles	23
1.5.3	Single-Bilayered Vesicles	24
	References	24

2	Curvature Elasticity of Fluid Membranes	26
2.1	Shape Problem in Red Blood Cell	26
2.1.1	Membranes in Cell	26
2.1.2	High Deformability of Cell Membranes	28
2.1.3	Difficulty in the Explanation of Discocyte Shape	29
2.2	Classic Differential Geometry of Surface	30
2.2.1	Lipid-Bilayer Vesicle Viewed as a Closed Surface	30
2.2.2	Space Curve	31
2.2.3	Surface and Parametric Curves	33
2.2.4	First Fundamental Form	33
2.2.5	Area	35
2.2.6	The Normal and the Tangent Plane	36
2.2.7	Second Fundamental Form	36
2.2.8	Christoffel Symbols	37
2.2.9	Curves and Directions on a Surface	38
2.2.10	Normal Curvature of a Curve on a Surface	40
2.2.11	Principal Directions, Line of Curvature and Principal Curvatures	41
2.2.12	The Mean Curvature and the Gaussian Curvature	42
2.3	Differential Invariants on a Surface	45
2.3.1	Gradient of a Scalar Field	45
2.3.2	Divergence of a Vector Field	47
2.3.3	Laplace-Beltrami Operator on a Scalar Function	48
2.3.4	Two-Dimensional Curl of a Vector	48
2.3.5	Other Differential Invariants	49
2.4	Curvature Elasticity of Fluid Membranes in Liquid Crystal Phase	50
2.4.1	Fluid Membranes Viewed as Liquid Crystals	50
2.4.2	Helfrich's Approach	51
2.4.3	A Derivation by Way of 2D Differential Invariants	56
2.4.4	The Spontaneous Curvature Viewed from Landau-de Gennes Theory	60
2.4.5	Discussion of Helfrich Bending Energy on Liquid Crystal Point of View	64
2.4.6	Spontaneous Curvature and Flexoelectric Effect	66

26	References	68
26	3 Shape Equation of Lipid Vesicles and Its Solutions	70
26	3.1 Mathematical Preliminary	70
28	3.2 General Shape Equation	81
29	3.3 Spherical Vesicles	86
30	3.4 Nearly Spherical Vesicles and Third Order Energy Variation	90
30	3.5 Circular Cylindrical Vesicles	94
31	3.6 Noncircular Cylindrical Vesicles	96
33	3.7 Clifford Torus	117
33	3.8 Dupin Cyclide	123
35	3.9 Shape Equation for Axisymmetric Vesicles	129
36	3.10 Circular Biconcave Discoid	138
36	3.11 Surfaces of Revolution with Constant Mean Curvature and Extended Surfaces	142
37	3.12 Challenge	148
38	References	148
40	4 Governing Equations for Open Lipid Membranes and Their Solutions	151
41	4.1 Mathematical Preliminary	151
42	4.1.1 Surface Theory Based on Moving Frame Method	151
45	4.1.2 Hodge Star, Stokes Theorem and some Important Geometric Relations	153
45	4.1.3 Variational Theory Based on the Moving Frame Method	155
47	4.2 Governing Equations for Open Lipid Membranes	162
48	4.2.1 Governing Equations	162
48	4.2.2 Compatibility Conditions	164
49	4.3 Analytic Solutions	165
50	4.4 Quasi-Exact Solutions	167
50	4.5 Challenge	168
51	4.5.1 Neck Condition of Two-Phase Vesicles in the Budding State	168
56	4.5.2 Minimal Surface with Boundary Curve	170
60	References	170
64	5 Theory of Tilted Chiral Lipid Bilayers	172
66	5.1 Theory of Tilted Chiral Lipid Bilayers with Strong Chirality	173
	5.1.1 Geometric Description and Free Energy	173

5.1.2	Tilt-Equilibrium and Surface-Equilibrium Equations in Case of Strong Chiral Effect	177
5.1.3	Wound Ribbon Helix	179
5.1.4	Twisted Ribbon	185
5.1.5	Spherical Vesicle	189
5.2	General Theory for TCLB	192
5.2.1	General Formula of Chiral Free Energy	192
5.2.2	The Effect of Other Elastic Constants	197
5.2.3	High- and Low-Pitch Helical Structures of TCLBs	201
5.3	Concise theory for Chiral Lipid Membranes	208
5.3.1	Brief Introduction to Moving Frame Method and Exterior Differential Forms	209
5.3.2	Constructing the Free Energy	212
5.3.3	Governing Equations to Describe Equilibrium Configurations of CLMs without Free Edges	213
5.3.4	Governing Equations to Describe Equilibrium Configurations of CLMs with Free Edges	215
5.3.5	Solutions to Governing Equations of CLMs without Free Edges	218
5.4	Challenge	231
	References	232
6	Some Untouched Topics	234
6.1	Nonlocal Theory of Membrane Elasticity	234
6.1.1	Area-difference Elasticity	234
6.1.2	Membrane with Nonlocal Interactions	236
6.2	Numerical Simulations	237
6.2.1	Surface Evolver	238
6.2.2	Phase Field	248
6.3	Effects of Membrane Skeleton	249
6.3.1	Composite Shell Model of Cell Membranes	250
6.3.2	Stability of Cell Membranes and the Function of Membrane Skeleton	252
6.4	Application in Elasticity of Low-Dimensional Carbon Materials	254
6.4.1	Revised Lenosky Model	254

6.4.2	Carbon Nanotubes	256
References		259
Appendix A	Tensor Calculus	262
Appendix B	The Gradient Operator	268
Reference		270
Appendix C	Elastic Theory of Membranes Viewed from Force Balance	271
Appendix D	A Different Viewpoint of Surface Tension	273
Reference		274

In this chapter, firstly we are going to give a general description of liquid crystalline states, states which are different from the commonly known three states of matter. The essential characteristic of liquid crystal, unlike ordinary liquid state, is that the constituent molecules although have no positional order but possess orientational order. Next, the two-component system (lipid/water) liquid crystal will be introduced. Then follows with discussions on the lipid/water system, especially the liquid crystalline state of biomembranes, mainly bilayers. Detailed discussion on the phase transitions in biomembranes will be introduced next. The part of biophysics of biomembrane or the molecular factors affecting the phase transitions in lipid bilayer will also be included in our discussion. Finally, a comprehensive introduction of the methods in preparing artificial vesicles will be given, which may be helpful for experiments in investigating the phase transitions of vesicles. The subject matter in this chapter is mainly taken from several well known books¹⁻³, especially many schematic figures are based on the book written by Dava.

1.1 Liquid Crystals

1.1.1 Mysterious Matter

Since 1960s there are several nice books which serve for teaching us a new state of matter—liquid crystal¹⁻³. In his famous book¹, Nobel Prize Laureate de Gennes remarked that liquid crystals are beautiful and mysterious. This state shows different feature from our common knowledge that matter exists in three states: solid, liquid, and gas. We are all exposed by the name