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The ancillary material for the book including a set of computer codes for a number of examples in the book can be found at <http://maranasgroup.com>. The solutions to exercises are available to instructors upon request.